An influential view of human cognition situates core components of natural language at the center of diverse domains of thought (Levinson, 2003; Whorf, 1940). Arithmetic reasoning is often seen in this light because it is plausible that the structured hierarchy found in algebraic expressions, such as $2 \times (5 - 3)$, is mentally constructed from syntactic routines underlying the interpretation of sentences, such as “The man saw the boy who kicked the ball.” In both cases, rules for evaluating subparts of the structure must be applied recursively in order to arrive at the semantic value of the whole. Hauser, Chomsky, and Fitch (2002) state that both language and number rely on a recursive computation that exploits the same neural mechanism operating over linguistic structures. Recursion, they suggest, evolved over time from a process that was highly domain specific to a process that was domain general, and this change gave humans the possibly unique ability to use recursion to solve nonlinguistic problems, notably, numerical manipulation. Likewise, Spelke and Tsivkin (2001) stated that natural language was the “most striking combinatorial system” of the human mind and that formal mathematics might be one of this system’s “richest and most dramatic outcomes” (p. 84).

The idea that nonlinguistic domains of thought, such as number, may co-opt the recursive machinery of language is made explicit by Chomsky (1998). He argued that the human faculty for arithmetical reasoning can be thought of as being abstracted from language and that it operates by “preserving the mechanisms of discrete infinity and eliminating the other special features of language” (p. 169). Similarly, Fitch, Hauser, and Chomsky (2005) state that the only clear demonstrations that recursion operates in human cognitive domains come from mathematical formulas and computer programming, which clearly employ the same reasoning processes that language does. This view has been sharpened by the proposal that the left inferior frontal gyrus (IFG) acts “supramodally” to forge complex hierarchical dependencies for nonlinguistic domains (Fadiga, Craighero, & D’Ausilio, 2009; Tettamanti & Weniger, 2006). The former authors propose that IFG and ventral premotor cortex “are tuned to detect and represent complex hierarchical dependencies, regardless of modality and use” (p. 448). The authors discuss this idea in the context of language, action, and music, because these three domains “share a common syntax-like structure” (p. 448). As discussed, however, this same syntax-like structure is also present in algebraic expressions, a characteristic that has been proposed to derive directly from the properties of natural language.
Therefore, if left IFG truly acts as a supramodal parser of hierarchical structure, this region should be equally involved in processing and manipulating the hierarchical dependencies of algebraic expressions.

It is worth noting that the latter hypothesis is neutral regarding the specific processes that are implemented in left IFG. Such processes might correspond to syntactic movement (Grodzinsky & Santi, 2008), registration of long-range dependencies (Friederici, Bahlmann, Heim, Schubotz, & Anwander, 2006), unification of lexical information (Hagoort, 2005), or selection among competing representations (Novick, Trueswell, & Thompson-Schill, 2010).

In the study reported here, we evaluated the role of language in arithmetic cognition by comparing the manipulation of linguistic versus algebraic expressions. To illustrate, consider the pair of statements labeled Argument A in Figure 1a. The statements are logically equivalent in the sense that the truth of the first statement guarantees the truth of the second one and vice versa. The two statements in Argument B are also logically equivalent. However, the two equivalence judgments rest on vocabulary from distinct cognitive domains. In Argument A, the judgment depends on whether the principal verb assigns the same semantic roles (i.e., who did what to whom) to X, Y, and Z across a syntactic transformation. In Argument B, the judgment depends on the properties of elementary algebraic operations (i.e., addition, subtraction) and relations (i.e., equality, inequality). Arguments C and D illustrate nonequivalent pairs of linguistic and algebraic statements, respectively.

It is uncontroversial that language mechanisms are required to encode the two statements of an argument. We tested the more substantive claim that language accompanies reasoning beyond the point of encoding in both the linguistic and algebraic domains. We obtained blood-oxygenation-level-dependent recordings from 21 right-handed volunteers with no known history of neurological disorders while they evaluated whether linguistic and algebraic arguments were logically equivalent (an equivalence task) and grammatically well formed (a grammar, or baseline, task). We first contrasted brain activity during the equivalence and grammar trials for each domain separately. We then compared activity during the linguistic and algebraic equivalence tasks to one another. If the syntactic routines of language underlie algebraic cognition, both equivalence tasks should activate left-hemisphere language regions. But in fact, although linguistic equivalence heavily recruited left fronto-temporal perisylvian regions (which are typically activated in language tasks; Devauchelle,}

![Diagram](image_url)
Oppeheim, Rizzi, Dehaene, & Pallier, 2009; Monti, Parsons, & Osherson, 2009; Pallier, Devauchelle, & Dehaene, 2011),
algebraic equivalence evoked less or equal activity in these
regions than the algebraic baseline task did. Instead, algebraic
equivalence recruited areas previously reported for number
cognition (Dehaene, Piazza, Pinel, & Cohen, 2003; Zago et al.,
2001).

Method
Participants

Twenty-one (12 female, 9 male) volunteers participated in the
experiment for monetary compensation after giving written
informed consent. The study was approved by the Cambridge
Local Research Ethics Committee. All subjects were right-
handed native English speakers with no history of neurologi-
cal disorder.

Stimuli

Stimuli for the equivalence tasks consisted of 64 arguments,
each of which contained a pair of statements (see Fig. 1a for
examples). Half of the arguments were linguistic, built around
ditransitive main verb and accompanying subject, direct
object, and indirect object. In half of the linguistic arguments,
the two statements in each pair were semantically equivalent,
differing only in voice (i.e., active vs. passive), use of clefting
(e.g., “It was . . .”), and other transformations that alter the
syntactic structure of a sentence but preserve its meaning (e.g.,
“Y gave X to Z” and “It was X that Y gave to Z”); in the other
half of the linguistic arguments, the two statements in each
pair were nonequivalent (e.g., “Y gave X to Z” and “Z was
given Y by X”). A given verb appeared in both equivalent and
nonequivalent linguistic arguments.

The other half of the arguments were algebraic, involving
arithmetic relations between three unknowns, X, Y, and Z. In
half the algebraic arguments, the two statements in each pair
were arithmetically equivalent (e.g., “Y is greater than Z
divided by X” and “X times Y is greater than Z”); in the other
half of the algebraic arguments, the two statements in each
pair were nonequivalent (e.g., “Y is greater than Z divided by
X” and “Z times Y is greater than X”). Algebraic statements
were presented in natural language and did not make use of
algebraic symbolism. Participants were thus encouraged
(without explicit instruction) to exercise their intuition rather
than apply mathematical expertise. In addition, this format
also made algebraic and linguistic arguments orthographically
more similar and equally reliant on linguistic decoding of
stimuli. Assignment of X, Y, and Z to each thematic role (in
linguistic arguments) and to each unknown quantity (in alge-
braic arguments) was randomized for each trial and each
participant.

Stimuli for the grammar (baseline) task included all 64
arguments used in the equivalence task plus 24 ungrammatical
arguments, 12 of which were linguistic (e.g., “What X gave Z
was to Y”) and 12 of which were algebraic (e.g., “Y is minus
X equals Z”). Ungrammaticality was produced by deletions,
intrusions, and reorderings. Trials with ungrammatical
arguments were not analyzed and served only to make the baseline
task credible.

Task

Each trial presented a single argument with the instruction to
perform either the equivalence or grammar task. In the equiva-
rence task, participants were required to assess whether the
two statements within the argument were semantically equiva-
 lent, that is, whether each implied the other. The grammar task
required participants to assess whether both statements were
grammatically well formed. In the latter task, participants
were instructed to disregard whether the two statements in an
argument were equivalent and to consider each statement
separately.

Design and procedure

Following an event-related design, we began each trial with a
2-s instruction cue displaying the single word “equivalence”
or “grammar” on the screen. Subsequently, the argument’s
first statement was presented alone for 3 s. Then the second
statement appeared, and the completed argument (both state-
ments) remained on screen for up to 23 s. After the subject’s
response (via a button box) or when 23 s had expired, the trial
was terminated, and the argument was replaced by a fixation
cross (see Fig. 1b). The duration of the ensuing fixation period
was jittered, with the interval drawn from an exponential dis-
tribution (M = 6,062 ms; range = 3,000–12,000 ms).

The 64 grammatically correct arguments were presented
across four scans. On each scan, 16 arguments of the same
type (linguistic or algebraic) were presented twice: once for an
equivalence judgment and once for a grammatical evaluation.
In addition, on each scan, 6 ungrammatical arguments were
randomly interspersed among the grammar trials (never
among equivalence trials). On half of the scans, participants
first assessed arguments for equivalence and then for gram-
mor; the reverse order was followed for the other half of
the scans (the order was counterbalanced for each subject).
Stimuli were presented using E-Prime software (Version 2;
Schneider, Eschman, & Zuccolotto, 2001). The procedure was
initiated by a scanner pulse at the beginning of each run to
ensure synchrony between stimulus onset and data acquisi-
tion. Timing files reporting the onset and offset of each event
were produced by E-Prime and used to create the regressors
for the functional MRI (fMRI) data analysis.

fMRI acquisition

Image data were acquired with a 3-T Siemens Tim Trio scan-
ner. T2*-sensitive images were acquired with a gradient echo
sequence (repetition time = 2,000 ms, echo time = 30 ms, flip angle = 78°, field of view = 192 × 192 mm) in 32 descending slices with a 3 mm³ voxel size and a 0.25 interslice distance factor. Structural images were acquired with a standard T1-sensitive magnetization-prepared rapid-acquisition gradient echo sequence with a 1 mm³ voxel resolution (repetition time = 2,250 ms, echo time = 2.99 ms, flip angle = 9°, field of view = 256 × 240 × 160 mm).

**fMRI data analysis**

Analyses were performed using Version 4.1.4 of FSL, the software library of the Oxford Centre for Functional MRI of the Brain (FMRIB; Smith et al., 2004). Prior to functional analyses, each individual echo-planar imaging time series was motion-corrected to the middle time point using a six-parameter, rigid-body method. Data were smoothed with a Gaussian kernel of 5 mm full-width half-maximum, and signal from extraneous nonbrain tissue was removed using the Brain Extraction Tool (part of the FSL library). Autocorrelation was corrected using a prewhitening technique. Statistical analyses were performed using an event-related general linear model approach, as implemented in the fMRI Expert Analysis Tool (part of the FSL library).

Equivalence and grammar trials (both for algebraic and linguistic arguments) were modeled with separate regressors. In each case, activations were modeled from the onset of the second statement until the subject’s response, thus accounting for different response time (RT) latencies and allowing interpretation of statistical parametric maps as reflecting activation per unit time. In addition, the cue period and the initial 3 s of each trial, in which only one statement was visible, were modeled with a single regressor that included both equivalence and grammar trials. Prior to multisubject analyses, each individual data set was coregistered to Brain Template 152 from the International Consortium for Brain Mapping (this template was originally created by the Montreal Neurological Institute) using 7- and 12-parameter optimization methods. Group-mean statistics for each contrast of interest were generated with a mixed-effects model resulting from the use of within-session variance (i.e., fixed effects) at the single-subject level and between-session variance (i.e., random effects) at the group level. Statistical parametric maps were computed in FMRIB’s Local Analysis of Mixed Effects (FLAME) 2 software; local activations were assessed with Threshold Free Cluster Enhancement (part of the FSL library) at a corrected significance level ($p < .05$).

For each participant, four contrasts were performed: (a) activation on equivalence trials greater than activation on grammar trials for linguistic arguments, (b) activation on equivalence trials greater than activation on grammar trials for algebraic arguments, (c) activation on linguistic equivalence trials greater than activation on algebraic equivalence trials, and (d) activation on algebraic equivalence trials greater than activation on linguistic equivalence trials. To filter out false activations, we restricted the first two contrasts to voxels with regression parameters greater than zero for either the equivalence or the grammar task, as compared with fixation. Likewise, the second two contrasts were limited to voxels with regression parameters greater than zero for at least one of the two contrasts of equivalence minus grammar trials. The interaction effect of task (equivalence, grammar) and argument type (linguistic, algebraic) was thus evaluated only within voxels that were responsive to at least one of the two equivalence tasks and not in voxels driven exclusively by the grammar trials. Furthermore, the use of these two masking procedures decreased the number of multiple comparisons performed, which made the analysis more sensitive.

Finally, we employed a region-of-interest (ROI) analysis to assess interactions in areas that have been previously discussed in the language literature (Monti et al., 2009; Pallier et al., 2011) and the numeracy literature (Dehaene et al., 2003; Zago et al., 2001). ROIs were defined on a purely structural basis using the Harvard-Oxford cortical atlas available with FSL. We focused on left IFG pars opercularis, triangularis, and orbitalis; the posterior sections of superior temporal gyrus (STG) and middle temporal gyrus (MTG); and angular gyrus (AG), as well as bilateral superior parietal lobule and the horizontal section of anterior intraparietal sulcus. First, the subject-wise regression coefficients were entered in a three-way analysis of variance (ANOVA) with task (equivalence, grammar), argument type (linguistic, algebraic), and ROI as within-subjects factors. As expected, there was a significant interaction among the three factors, which indicated that there was a different Task × Argument Type interaction across ROIs, $F(9, 180) = 7.88, p < .05$. Following this analysis, we conducted separate two-way ANOVAs, one per ROI, including task and argument type as within-subjects factors. Finally, to assess the simple effect of inference across argument types, we conducted follow-up analyses on significant Task × Argument Type interactions using paired-sample $t$ tests.

**Results**

**Behavioral performance**

In the equivalence task, linguistic arguments were assessed with greater accuracy, $t(20) = 3.87, p < .05$, and speed, $t(20) = 4.04, p < .05$, than were algebraic arguments. Linguistic equivalence trials lasted an average of 9.76 s ($SD = 2.13$ s), and participants had a mean accuracy of 84.07% ($SD = 7.84$%); algebraic equivalence trials lasted an average of 11.05 s ($SD = 2.43$ s) and were judged with a mean accuracy of 78.13% ($SD = 3.95$%). Grammaticality judgments did not significantly differ in accuracy across linguistic arguments ($M = 90.62\%$) and algebraic arguments ($M = 95.31\%$), $t(20) = 1.87, p > .05$. Linguistic grammar trials lasted an average of 5.96 s ($SD = 0.68$ s), and algebraic trials lasted an average of 5.46 s ($SD = 0.74$ s), a small but significant difference, $t(20) = 3.72, p < .05$. 

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Functional brain activity

In relation to grammar trials with linguistic stimuli, equivalence trials with linguistic stimuli elicited greater activation in left-hemisphere regions previously reported for linguistic processing (Fig. 2a; Monti et al., 2009; Pallier et al., 2011). In particular, this contrast revealed significant activation in both the pars opercularis and pars triangularis of left IFG (Brodmann’s area, BA, 44 and 45, respectively), as well as in posterior segments of MTG (BA 21) and STG (BA 22). Additional activations were detected in left middle frontal gyrus (BA 6), superior parietal lobule (SPL; BA 7), AG of inferior parietal lobule (IPL; BA 39), the horizontal segment of anterior intraparietal sulcus (hIPS), medial precuneus (BA 7), and right IPL (BA 7).

Crucially, when the same comparison was performed for algebraic stimuli, no significant activation was detected in any of the perisylvian language regions (Fig. 2a). Instead, extensive activation was observed in regions previously reported for number cognition and calculation (Dehaene et al., 2003; Zago et al., 2001), including bilateral hIPS and SPL, left IPL (BA 40), middle frontal gyrus (BA 6), and the medial segments of precuneus (BA 7).

Next, to compare the two tasks quantitatively, we directly contrasted language equivalence with algebraic equivalence. This contrast confirmed our preceding analyses of equivalence and grammar trials (Fig. 2b). Subtraction of algebraic equivalence trials from linguistic equivalence trials revealed activations in a set of areas in left inferior frontal and temporal regions that have been reported to process abstract syntactic frames and semantic constituents (Pallier et al., 2011). These activations were localized to left IFG pars opercularis, triangularis, and orbitalis (BA 47); posterior STG (BA 22); posterior and anterior MTG (BA 21); and AG and supramarginal gyrus (BA 22). However, the reverse subtraction of linguistic equivalence from algebraic equivalence revealed extensive activity in the bilateral hIPS, SPL, IPL, and right superior frontal gyrus.

Fig. 2. Results for the four main contrasts. The brain maps in (a) show areas of activation for the equivalence trial > grammar trial contrasts for linguistic and algebraic materials. The brain maps in (b) show areas of activation for the linguistic equivalence trial > algebraic equivalence trial contrast and the algebraic equivalence trial > linguistic equivalence trial contrast.
(BA 6). These regions have been linked to representation of quantity, Arabic numerals, and calculation (Dehaene et al., 2003; Zago et al., 2001; see Fig. 2b). Tables S1 to S4 in the Supplemental Material available online provide a detailed list of activations for each of these contrasts.

Finally, we conducted an ROI analysis on anatomically defined areas identified in previous literature, separately for language (Monti et al., 2009; Pallier et al., 2011) and number (Dehaene et al., 2003; Zago et al., 2001). The purpose of this analysis was to directly test the view that regions underlying linguistic competence participate in nonlinguistic cognition. The analysis confirmed the dissociation in neural activity for linguistic and algebraic tasks observed at the full brain level. As Figure 3 shows, IFG pars triangularis and pars orbitalis, STG, and MTG all exhibited the expected Task (equivalence, grammar) × Argument Type (linguistic, algebraic) interaction (this interaction was also marginally significant in IFG pars opercularis). For all of these regions, the interaction was driven by a significant simple effect of task for the linguistic stimuli only (this interaction was only marginally significant in STG). The converse interaction was also observed, as expected, in both right lateralized ROIs, anterior intraparietal sulcus, and SPL, for algebraic stimuli. A similar pattern was observed in the left lateralized ROIs (anterior intraparietal sulcus and SPL) for algebraic stimuli. Each analysis showed a significant simple effect of task in the case of algebraic stimuli, but in neither was the interaction significant. Finally, no interaction was observed in left AG, in which the simple effect of task was significant in the cases of both linguistic and algebraic stimuli, but the effect for algebraic stimuli only was driven by lower activation during the grammar trials than during the equivalence trials.

**Discussion**

Our findings indicate that beyond initial reading and comprehension of stimuli, the neural substrate of language does not intervene in algebraic reasoning. In fact, as Figure 3 shows, many of the perisylvian linguistic regions appeared to be less active during the algebraic equivalence task than during simple reading (i.e., the grammar task), a finding that parallels previous reports in the domain of mental calculation (Fodor-enko, Behr, & Kanwisher, 2011; Zago et al., 2001). Interpretation of this finding will require further research. In contrast, perisylvian regions were observed to be active for the linguistic equivalence task, as expected.

The observed dissociation is consistent with the findings of investigations of linguistic and arithmetic abilities in neurological patients (Cappelletti, Butterworth, & Kopelman, 2001; Cipolotti, Butterworth, & Denes, 1991) and individuals with developmental disorders (Butterworth, 2005; Mazzocco et al., 2006). Particularly relevant is the case of patients with agrammatic aphasia, who exhibit intact understanding of the rules, structure, and operations of abstract algebra but perform at chance levels in standard assessments of language (Varley, Klessinger, Romanowsky, & Siegal, 2005). Our results suggest that the latter finding is not due to compensatory strategies or functional remapping following brain injury.

Our linguistic and algebraic problems yielded common activations in left posterior and medial regions of parietal cortex. In the domain of number cognition, posterior SPL has been proposed to support mental arithmetic (Knops, Thirion, Hubbard, Michel, & Dehaene, 2009) by mediating attentional orientation along the spatial representation of the number line (Dehaene et al., 2003). However, such spatial processing in SPL does not seem to be specific to number cognition because it occurs across a variety of tasks requiring manipulation and rearrangement of information in working memory (Koenigs, Barbey, Postle, & Grafman, 2009) and allocation of attention (see Dehaene et al., 2003). In addition, the engagement we found here in left SPL is consistent with its recruitment across verbal (Osaka, Kemori, Morishita, & Osaka, 2007), numeric, and spatial working memory tasks (Hanakawa et al., 2002), as well as for linguistic and logic inference (Monti et al., 2009). Finally, the common recruitment of precuneal cortex across linguistic and algebraic materials may reflect the generally greater complexity of the equivalence tasks, as compared with the grammar tasks, across both domains (cf. Wallentin, Roepstorff, Glover, & Burgess, 2006).

Algebraic problems strongly and specifically recruited bilateral portions of hIPS. These regions are typically activated in the context of numerical comparisons, approximation, and estimation (Nieder & Dehaene, 2009; Piazza, Pinel, Le Bihan, & Dehaene, 2007). Our stimuli, in contrast, contained neither numerals nor specific magnitudes, yet the foci uncovered by the equivalence task for algebra closely matched the hIPS subregions defined by a large meta-analysis of neuroimaging studies on number cognition (Dehaene et al., 2003). A potential interpretation is that hIPS embodies a domain-general ordering ability extending not just to numbers but also to algebraic variables (as we found here), letters of the alphabet (Fias, Lammertyn, Caessens, & Orban, 2007), months of the year (Ischebeck et al., 2008), abstract relata involved in transitive reasoning (Prado, Noveck, & Van Der Henst, 2010), and other high-order cognitive sequences (Jubault, Ody, & Koechlin, 2007).

A rival interpretation of the hIPS activation seen here is that participants mentally replaced algebraic variables with numerals to solve the equivalence trials. This strategy, however, is prone to error because it does not allow participants to unambiguously order the three variables according to their magnitude; the accurate performance seen in the equivalence task thus argues against a number-substitution interpretation. We note that the use of hIPS might extend to ordering the constituents of complex sentences, which would explain the appearance of IPS (on the left side) among the regions revealed by our language contrast (and also in our earlier work on logical reasoning; Monti et al., 2009). More research is needed on how the brain, and left and right IPS in particular, represents cardinal versus ordinal information (Cohen Kadosh, Muggleton, Silvanto, & Walsh, 2010; Nieder & Dehaene, 2009; Zago et al., 2008).
Fig. 3. Results of the region-of-interest analysis of brain areas associated with language processing and arithmetic cognition in the literature. General linear model (GLM) regression coefficients are shown as a function of trial type. Significant differences between trial types are indicated with an asterisk (* p < .05), and marginally significant differences between trial types are indicated with a dagger († 0.05 < p < .08). Error bars indicate standard errors. IFG op = inferior frontal gyrus pars opercularis, AG = angular gyrus, STG = superior temporal gyrus, MTG = middle temporal gyrus, aIPS = anterior intraparietal sulcus, SPL = superior parietal lobule, IFG orb = inferior frontal gyrus pars orbitalis, IFG tri = inferior frontal gyrus pars triangularis.
On the behavioral level, algebraic equivalence was more difficult than linguistic equivalence, as shown both by RTs and error rates. If linguistic processes were involved in algebra, we might therefore expect more perisylvian activation for algebra equivalence trials minus algebra baseline trials than for language equivalence trials minus language baseline trials. Yet the reverse was true: Algebra uncovered no perisylvian activity beyond what was needed for initial encoding, whereas abundant perisylvian activity was observed for language. This pattern is consistent with the hypothesis of the independence of algebraic reasoning from linguistic processes (subsequent to reading).

In summary, our data exhibit a neural dissociation between the syntax-like operations of algebra and those of natural language. In our adult volunteers, algebraic operations did not recruit any more language resources than did simple reading (i.e., the grammar task); in contrast, they did rely on areas previously linked to arithmetic cognition. These results are consistent with neuropsychological evidence (Butterworth, 2005; Cipolotti et al., 1991; Varley et al., 2005) and findings showing parallel dissociations between the operations of language and those of logical reasoning (Monti & Osherson, 2012; Monti, Osherson, Martinez, & Parsons, 2007; Monti et al., 2009). The present results constitute evidence against the view that language forms the basis of structured thought across cognitive domains (Fadiga et al., 2009; Fitch et al., 2005; Tettamanti & Weniger, 2006).

Declaration of Conflicting Interests
The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

Supplemental Material
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